

Data Structures TD3: Trees

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October 2023

INFORMATION: We will correct the following exercises today:

- All the exercises from the introduction section;
- In section 2, exercise 1;
- In section 3, exercise 1;

The other ones are for you to practice, if you want to.

1 Introduction

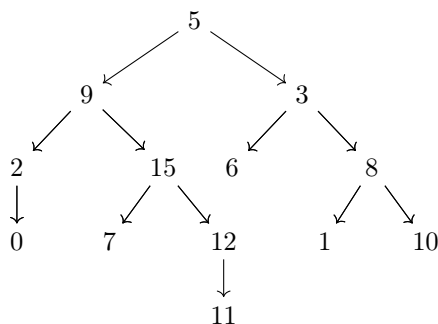
1. **Research algorithm** (the one from the CM):

```
Require:  $T, value$   
1:  $S \leftarrow S_0$   
2:  $add(T, S)$   
3: while  $!isEmpty(S)$  do  
4:    $tree \leftarrow get(S)$   
5:    $remove(S)$   
6:   if  $!isEmpty(tree)$  then  
7:     if  $tree[‘root’] == value$  then  
8:       return  $true$   
9:     end if  
10:    if  $!isEmpty(tree[‘right child’])$  then  
11:       $add(tree[‘right child’], S)$   
12:    end if  
13:    if  $!isEmpty(tree[‘left child’])$  then  
14:       $add(tree[‘left child’], S)$   
15:    end if  
16:  end if  
17: end while  
18: return  $false$ 
```

Apply the algorithm and write down the different states of the stack S after each iteration of the while loop:

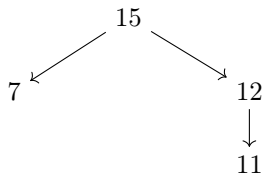
(a) for the following tree and the value 13;

(b) for the following tree and the value 15;



2. In the tree (let it be T) from the previous question:

- What is the value of $T[\text{'left child'}][\text{'right child'}][\text{'right child'}][\text{'root'}]$?
- What is the value of $T[\text{'right child'}][\text{'left child'}][\text{'left child'}]$?
- What is the value of $T[\text{'right child'}]$?
- What is the value of the root of $T[\text{'left child'}][\text{'left child'}][\text{'left child'}]$?
- What is the value of $T[\text{'right child'}][\text{'right child'}][\text{'left child'}][\text{'root'}]$?
- What is the value of $T[\text{'left child'}][\text{'left child'}]$?
- How do you access the node of value 6?
- How do you access the node of value 7?
- How do you access the node of value 11?
- How do you access the following subtree?



- How do you change the value of the node of value 8 to 4 ?

2 Reading an Algorithm

- What does the following algorithm do? *Hint: You can draw a small tree and apply the algorithm to it to get an idea of what it does.*

```

Require:  $T$ 
1: if  $!isEmpty(T)$  then
2:    $Q \leftarrow Q_0$ 
3:    $add(T, Q)$ 
4:    $c \leftarrow 0$ 
5: else
6:   return 0
7: end if
8: while  $!isEmpty(Q)$  do
9:    $tree \leftarrow get(Q)$ 
10:   $remove(Q)$ 
11:   $c \leftarrow c + tree[\text{'root'}]$ 
12:  if  $!isEmpty(tree[\text{'left child'}])$  then
13:     $add(tree[\text{'left child'}], Q)$ 
14:  end if
15:  if  $!isEmpty(tree[\text{'right child'}])$  then
16:     $add(tree[\text{'right child'}], Q)$ 
17:  end if
18: end while
19: return  $c$ 
  
```

- What does the following algorithm do? *Hint: You can draw a small tree and apply the algorithm to it to get an idea of what it does.*

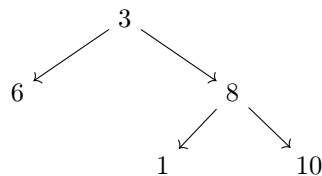
```

Require:  $T$ 
1: if  $!isEmpty(T)$  then
2:    $Q \leftarrow Q_0$ 
3:    $add(T, Q)$ 
4:    $c \leftarrow 0$ 
5: else
6:   return 0
7: end if
8: while  $!isEmpty(Q)$  do
9:    $tree \leftarrow get(Q)$ 
10:   $remove(Q)$ 
11:   $c \leftarrow c + 1$ 
12:  if  $!isEmpty(tree['left child'])$  then
13:     $add(tree['left child'], Q)$ 
14:  end if
15:  if  $!isEmpty(tree['right child'])$  then
16:     $add(tree['right child'], Q)$ 
17:  end if
18: end while
19: return  $c$ 

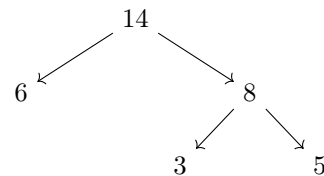
```

3 Creating an Algorithm

- Describe an algorithm that would verify if all the nodes of a tree are different pairwise (*i.e.*, the same value never appears twice). Describe the input(s), output(s), and what this algorithm would do.
- Describe an algorithm that would verify if a tree is a *sum tree*. What we call a sum tree here is a tree where each node is equal to the sum of its direct children. The leafs (*i.e.*, the nodes where both the left and right children are empty) can have any values. For example below, (a) is not a sum tree but (b) is.



(a) Not a sum tree



(b) Sum tree ($3 + 5 = 8$ and $6 + 8 = 14$)

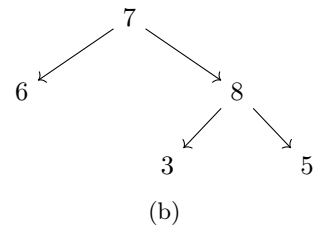
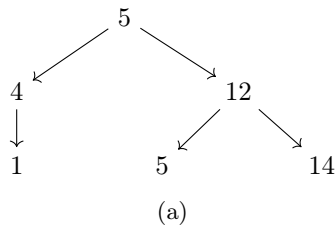
4 Optional Exercise - Due December 8th

```

Require:  $T$ 
1: if  $!isEmpty(T)$  then
2:    $Q \leftarrow Q_0$ 
3:    $add(T, Q)$ 
4: else
5:   return  $True$ 
6: end if
7: while  $!isEmpty(Q)$  do
8:    $tree \leftarrow get(Q)$ 
9:    $remove(Q)$ 
10:  if  $!isEmpty(tree['left child'])$  then
11:    if  $tree['left child']['root'] > tree['root']$  then
12:      return  $False$ 
13:    end if
14:     $add(tree['left child'], Q)$ 
15:  end if
16:  if  $!isEmpty(tree['right child'])$  then
17:    if  $tree['right child']['root'] < tree['root']$  then
18:      return  $False$ 
19:    end if
20:     $add(tree['right child'], Q)$ 
21:  end if
22: end while
23: return  $True$ 

```

1. Apply the algorithm above to the trees (a) and (b);



2. What does this algorithm do? *Hint: you can try on other trees to get a better idea.*